Title: Logarithms

Brief Overview:

In this Concept Development Unit, the concept of logarithms is discussed. The relationship between exponential equations and logarithmic equations is explored. The properties of logs are discussed and applied. Students will practice the new skills using cooperative learning games and real–world applications.

NCTM Content Standard/National Science Education Standard:

- understand relations and functions and select, convert flexibly among, and use various representations for them
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions

Grade/Level:

Grades 9 – 12/Algebra 2, College Algebra, Pre–Calculus

Duration/Length:

Three 90-minute class periods

Student Outcomes:

Students will:

- Rewrite exponential expressions in logarithmic form and vice versa
- Apply properties of logarithms
- Solve exponential and logarithmic equations

Materials and Resources:

- Index cards
- Graphing calculators
- Worksheets
    - Concentration Game with Logarithms
    - Basic Logarithms Flash Cards
    - Advanced Logarithms Card Game
    - Reteaching Properties of Logarithms
    - Laws of Logarithms
    - Basic Exponential and Logarithmic Equations Round Table Game
    - Laws of Logarithms in Logarithmic Equations Coach/Player Game
    - Solving Exponential and Logarithmic Equations
    - Applications of Logarithms

Development/Procedures:

Lesson 1

Preassessment – Students are expected to know how to graph functions and read values off of the graph. They should also be able to solve polynomial equations and be familiar with the properties of exponents and radicals. They have already had a lesson on *e*.

Distribute graphing calculators. Assess the students' ability to apply these skills by posing the following question:

Five students catch the flu at the end of winter break. They come to school contagious after New Year's Day. The spread of the virus can be modeled by $N = 5 + 2.25^t$, where N is the number of people infected after t days.

a) Graph the function using your calculator.

b) Find how many students will be infected after 4 days? Answer: 30

c) The school will close if 300 students are sick. How many days after the winter break will this occur? Answer: 7

Launch – In the previous example, data values can be easily read off of the graph or table of the function. Ask students to algebraically check the answer to part (b). Now invite students to solve part (c) algebraically. Students will run into the difficulty of how to solve for the *t*. Ask students what strategy they think might help them solve for the *t*. Possible student responses at this point might be dividing by 2.25 or taking an x^{th} root.

$$300 = 5 + 2.25^t$$
$$295 = 2.25^t$$

Emphasize that what the students are trying to figure out is to what power they have to raise 2.25 in order to get 295.

Teacher Facilitation – Look at a simpler example that could be solved by "guess and check".

What does x have to equal in the problem $3^x = 9$?

Students should quickly be able to say that $x = 2$.

Introduce the idea of rewriting the exponential expression in logarithmic form. To solve $3^x = 9$ for *x*, rewrite it in logarithmic form: $\log_3 9 = x$.

Practice rewriting several exponential expressions as logarithms and vice versa with the entire class:

$2^5 = 32 \rightarrow \log_2 32 = 5$

$\log_{16} 4 = \frac{1}{2} \rightarrow 16^{\frac{1}{2}} = 4$

$3^{-1} = \frac{1}{3} \rightarrow \log_3 \frac{1}{3} = -1$

$\log_{25} 5 = \frac{1}{2} \rightarrow \sqrt{25} = 5$

$\log_3 81 = m \rightarrow 3^m = 81$ Ans: 4

$8^y = 2 \rightarrow \log_8 2 = y$ Ans: $\frac{1}{3}$

$\log_2 \frac{1}{4} = w \rightarrow 2^w = \frac{1}{4}$ Ans: –2

$36^a = 6 \rightarrow \log_{36} 6 = a$ Ans: $\frac{1}{2}$ ½

Student Application – Have the students practice writing exponential equations into logarithmic equations by playing the "Concentration Game with Logarithms."

Embedded Assessment – Observe student input during practice problems to assess whether they understand the concept. Students will be helping each other to get the correct matches during the game. During the card game, the teacher circulates through the room and assists students if they need help. Students can be asked to write down the pairs they created and turn in the paper to the teacher after the game to assess their understanding.

Reteaching/Extension –

- Use "Basic Logarithm Flashcards" that gradually increases in difficulty for students who need reteaching.
- Use the "Advanced Logarithms Card Game" for students who understand the concept. Students who understand the concept can also work together to make a new set of game cards.

Lesson 2

Preassessment – Use 3 – 5 of the concentration game cards from the previous lesson (some exponential and some logarithmic) and have students write the opposite form.

Launch – Have students examine the buttons on a scientific or graphing calculator to determine which logs can be done on the calculator. Ask students to try to evaluate the following logs to get the correct answer on the calculator with those logarithm buttons:

$$\ln 15 = 2.708$$

$$\log_{10} 1000 = 3$$

$$\log_5 125 = 3$$

Students should be able to complete the first 2 examples, but will ask how to enter the bases since the calculator only has a log button. They will not be able to get the calculator to evaluate the last example.

Teacher Facilitation – Discuss that the calculator can only evaluate log base *e* (which is also called natural log and uses the notation *ln*) and log base 10 (which is called common log and uses the notation *log*).

Discuss the change of base formula, which allows you to change any base into the two bases the calculator can evaluate:

$$\log_a b = \frac{\log b}{\log a} = \frac{\ln b}{\ln a}$$

Now students will be able to change the base of the third example from the Launch and evaluate it:

$$\log_5 125 = \frac{\log 125}{\log 5} = \frac{\ln 125}{\ln 5} = 3$$

Instruct the students to evaluate the numerator and denominator separately and then divide. Have them try both the common log and the natural log, so that they see that the intermediate results will look differently, but the final answer will be the same. This will help them understand that they can choose any base.

Discuss the other three properties of logarithms and relate them to the properties of exponents that students should already know. Consider placing the properties on sentence

strips or poster paper to hang around the classroom, leaving them up for future reference.

Product Property:
Concept: $\log ab$ can be rewritten as $\log a + \log b$
Compare to: $x^a \cdot x^b = x^{a+b}$

Quotient Property:
Concept: $\log \frac{c}{d}$ can be rewritten as $\log c - \log d$
Compare to: $\frac{x^a}{x^b} = x^{a-b}$

Power Property:
Concept: $\log m^n$ can be rewritten as $n \log m$
Compare to: $(x^a)^b = x^{a \cdot b}$

Practice rewriting several logarithmic expressions using the properties (both expanding and collapsing):

$\log_3 45$ → Change of base: $\frac{\log 45}{\log 3}$

$\log 5xy$ → Product Property: $\log 5 + \log x + \log y$

$\ln \frac{3}{7}$ → Quotient Property: $\ln 3 - \ln 7$

$\log w^4$ → Power Property: $4 \log w$

$\log_3 12x$ → Product Property and Change of Base: $\log_3 12 + \log_3 x = \frac{\log 12}{\log 3} + \frac{\log x}{\log 3}$

$\ln x^2 + \ln y^3 = 2\ln x + 3\ln y$ → Product Property and Power Property: $\ln x^2 y^3$

$\log \frac{w^4 z^7}{25}$ → Quotient, Product, and Power Properties:
$\log w^4 z^7 - \log 25 = \log w^4 + \log z^7 - \log 2 = 4\log w + 7\log z - \log 2$

Student Application – All students will participate in the "Concentric Circles" activity to practice the properties.

Embedded Assessment – Observe student input during practice problems. Students will be helping each other to get the correct answers during the "Concentric Circle" game.

During the card game, stand in the middle of the circles and assist where necessary.

Reteaching/Extension –

- Use "Reteaching Properties of Logarithms" that gradually increases in difficulty for students who need reteaching.
- Use the "Laws of Logarithms" for students who understand the concept. Students who understand the concept can also work together to make a new set of game cards.

Lesson 3

Preassessment – Use 3 – 5 of the concentric circle game cards from the previous lesson and instruct students to write either the expanded or collapsed form of the expression.

Launch – Return to the flu example from Lesson 1. Ask students to solve part (b) by rewriting it in logarithmic form and using the properties of logs.

$$300 = 5 + 2.25^t$$

$$295 = 2.25^t$$

$$\log_{2.25} 295 = t$$

$$\frac{\log 295}{\log 2.25} = t$$

$$t = 7.01$$

Students have just solved an exponential equation by using the skills they have learned over the past two lessons.

Teacher Facilitation – To solve $3^x = 81$, we have been rewriting it in logarithmic form: $\log_3 81 = x$. Ask students what they relationship they notice between the base of the power and the base of the logarithm. Continue by asking what $\log_3 3$ is. Point out that instead of remembering in which order to rewrite exponential equations as logarithmic equations, we could instead do the following:

$$3^x = 81$$

$$\cancel{\log_3 3}^x = \log_3 81$$

The log base 3 "undoes" the base of 3 in the exponential expression, so that only *x* is left. Remind students that whatever they do to the left side of the equation, they also have to do to the right. Then finish up the example using the change–of–base property.

$x = \log_3 81$

$x = \dfrac{\log 81}{\log 3}$

$x = 4$

Sometimes, another way to solve an equation of this nature is to get the same base on both sides of the equation. In this case, $81 = 3^4$ so we can rewrite the left side:

$3^x = 81$

$3^x = 3^4$

Now we can take the log base 3 of both sides very easily:

$\cancel{\log_3 3}^x = \cancel{\log_3 3}^4$

$x = 4$

Use these practice problems to reinforce the idea of inverse operations:

1.

$5^{x+2} = 125$

$\cancel{\log_5 5}^{x+2} = \log_5 125$

$x + 2 = \dfrac{\log 125}{\log 5}$

$x + 2 = 3$

$x = 1$

Alternate method:

$5^{x+2} = 5^3$

$\cancel{\log_5 5}^{x+2} = \cancel{\log_5 5}^3$

$x + 2 = 3$

$x = 1$

2. $\log_7 3x = 2$

$\cancel{7}^{\cancel{\log_7} 3x} = 7^2$

$3x = 49$

$x = \dfrac{49}{3}$

3.

$$\sqrt{7}^{6x} = 65$$
$$7^{0.5(6x)} = 65$$
$$\cancel{\log_7 7}^{0.5(6x)} = \log_7 65$$
$$3x = \frac{\log 65}{\log 7}$$
$$x = 0.715$$

4.

$$\ln 3 + 2\ln f = 12$$
$$\ln 3 + \ln f^2 = 12$$
$$\ln 3f^2 = 12$$
$$\cancel{e}^{\cancel{\ln} 3f^2} = e^{12}$$
$$3f^2 = e^{12}$$
$$f^2 = \frac{e^{12}}{3}$$
$$f = \sqrt{\frac{e^{12}}{3}}$$
$$f = \frac{e^6}{\sqrt{3}} = \frac{e^6\sqrt{3}}{3}$$

Student Application – –Play the "Basic Exponential and Logarithmic Equations Round–Table Game", which practices exponential and logarithmic equations and the "Laws of Logarithms in Logarithmic Equations Coach/Player Game", which practices more difficult equations that also involve the properties of logs.

Embedded Assessment – In the "Basic Exponential and Logarithmic Equations Round–Table Game", students check each other's work and help each other. In the "Laws of Logarithms in Logarithmic Equations Coach/Player Game", students have to verbalize what they are steps they

are taking and help each other. Both of these activities can be collected by the teacher to assess student understanding.

Reteaching/Extension

- Use “Solving Exponential and Logarithmic Equations” that gradually increases in difficulty for students who need reteaching.
- Use the “Applications of Logarithms” for students who understand the concept to practice real–world application of logarithmic and exponential equations.

Summative Assessment:

An assessment of this unit should include problems involving the properties of logarithms and solving logarithmic and exponential equations. The assessment could include brief constructed response or extended constructed response items similar to the problems in Advanced Practice 3. In these questions, students could be asked to analyze or justify their results. This unit did not explicitly discuss exponential growth or decay, which are topics that could be covered either before or after this unit.

Authors:

Name Lana Conte
School Garrison Forest School

Name Miriam Snare
School Mercy High School

Concentration game with logarithms.

Rules of the game: Students should be in groups of 2–4 players. The deck is shuffled and all cards are laid out on the table face down. The first player turns over two cards. If they are a matching pair of logarithmic and exponential forms, the player keeps the pair. If the cards do not match, the player turns them back over and leaves them on the table. If the first player makes a match, s/he may take another turn. When no match is made, it is the next players turn. Students should be observing and checking each others' pairs to ensure that correct matches are made. They can coach each other in the process.
Alternative play: All cards can begin face up and students alternate picking up a matching pair. Coaching can occur between students. If the game begins with all cards face–down, students could leave the cards that have been viewed face–up for easier matching.

Values for Cards:

$\text{Log}_{27} 1 = x$	$27^x = 1$	$\log_3 \frac{1}{3} = x$	$3^x = \frac{1}{3}$	$\log_e 15 = x$
$\log_5 125 = x$	$5^x = 125$	$\log_5 \frac{1}{125} = x$	$5^x = \frac{1}{125}$	$e^x = 15$
$\log_{10} 1000 = x$	$10^x = 1000$	$\log_3 27 = x$	$3^x = 27$	$\log_{\frac{1}{2}} 8 = x$
$\log_{10} \frac{1}{100} = x$	$10^x = \frac{1}{100}$	$\log_4 2 = x$	$4^x = 2$	$\left(\frac{1}{2}\right)^x = 8$
$\log_{27} \frac{1}{3} = x$	$27^x = \frac{1}{3}$	$\log_4 64 = x$	$4^x = 64$	$\log_{16} 2 = x$
$\log_{\frac{1}{3}} 27 = x$	$\frac{1}{3}^x = 27$	$\log_2 \frac{1}{16} = x$	$2^x = \frac{1}{16}$	$16^x = 2$

Basic Logarithms Flashcards

The difficulty of questions increases to gradually develop patterns and increase students' understanding.

$2^{what?} = 4$	2	$2^{what?} = 8$	3	$3^{what?} = \sqrt{3}$	$\frac{1}{2}$
$\text{Log}_2 4$	2	$\text{Log}_2 8$	3	$\text{Log}_3 \sqrt{3}$	$\frac{1}{2}$
$\text{Log}_2 \frac{1}{2}$	–1	$\text{Log}_3 27$	3	$\text{Log}_3 \frac{1}{9}$	–2
$\text{Log}_2 2$	1	$\text{Log}_2 1$	0	$\text{Log}_2 \sqrt{2}$	$\frac{1}{2}$
$\text{Log}_2 \sqrt{8}$	$\frac{3}{2}$	$\text{Log}_2 \frac{1}{4}$	–2	$\text{Log}_2 \sqrt[3]{2}$	$\frac{1}{3}$
$4^{\text{what?}} = 2$	$\frac{1}{2}$	$4^{\text{what?}} = \frac{1}{4}$	–1	$4^{\text{what?}} = 8$	$\frac{3}{2}$
$\text{Log}_4 2$	$\frac{1}{2}$	$\text{Log}_4 \frac{1}{4}$	–1	$\text{Log}_4 8$	$\frac{3}{2}$
$\text{Log}_4 \sqrt{2}$	$\frac{1}{4}$	$\text{Log}_4 \frac{\sqrt[3]{4}}{4}$	$-\frac{2}{3}$	$\text{Log}_4 \frac{1}{2}$	$-\frac{1}{2}$
$\text{Log}_3 \frac{1}{27}$	–3	$\text{Log}_3 \frac{1}{\sqrt{27}}$	$-\frac{3}{2}$	$\text{Log}_3 \frac{\sqrt{27}}{\sqrt{3}}$	1
$\text{Log}_9 3$	$\frac{1}{2}$	$\text{Log}_9 \frac{1}{3}$	$-\frac{1}{2}$	$\text{Log}_9 27$	$\frac{3}{2}$

Advanced Logarithms Card Game

*The difficulty of the problems is adjusted for students who feel comfortable with the basics.

Rules of the game: 2–4 players in a group. The deck is shuffled and each player is dealt 5 cards from the deck. The deck is then placed face down on the table and the top card is flipped face up. The first player tries to match the value on the open card with a card in their hand. If they have a match, they draw a card from the deck and flip the top card in the deck face up. If there is no match, next player takes the turn. If nobody has a match, the next card in the deck is flipped. When all the cards in the deck have been flipped over, turn the entire deck face–down again and continue. The game continues until all the cards on the table are matched. Whoever has the most matched pairs by the end of the game, wins.
Alternative play: Same initial setup. When the top card in the deck is flipped, the player who matches the card in their hand the **fastest** takes the open card.
A match can be a numerical value on the card, or a logarithmic expression of the same value.

Questions and Answers for Cards:

$\mathbf{Log\,1}$	$\mathbf{0}$	$\mathbf{Log\sqrt{10}}$	$\mathbf{\frac{1}{2}}$	$\mathbf{Log\frac{1}{\sqrt{10}}}$	$\mathbf{-\frac{1}{2}}$
$\mathbf{Log\frac{1}{\sqrt[3]{100}}}$	$\mathbf{-\frac{2}{3}}$	$\mathbf{Log\frac{10}{\sqrt[3]{100}}}$	$\mathbf{\frac{1}{3}}$	$\mathbf{Log_2\sqrt{8}}$	$\mathbf{\frac{3}{2}}$
$\mathbf{Log_2\sqrt[3]{16}}$	$\mathbf{\frac{4}{3}}$	$\mathbf{Log_4 2}$	$\mathbf{\frac{1}{2}}$	$\mathbf{Log_4 8}$	$\mathbf{\frac{3}{2}}$
$\mathbf{Log_4\sqrt{32}}$	$\mathbf{\frac{5}{4}}$	$\mathbf{Log_4\frac{1}{2}}$	$\mathbf{-\frac{1}{2}}$	$\mathbf{Log_4\frac{\sqrt[3]{4}}{2}}$	$\mathbf{-\frac{1}{6}}$
$\mathbf{Log_3\frac{\sqrt{27}}{\sqrt{3}}}$	$\mathbf{1}$	$\mathbf{Log_{287}1}$	$\mathbf{0}$	$\mathbf{Log_4(-2)}$	**DNE**
$\mathbf{Log_{-2}\frac{1}{2}}$	**DNE**	$\mathbf{Log_4 0}$	**DNE**	$\mathbf{Log_{1/2}2}$	$\mathbf{-1}$
$\mathbf{Log_{1/2}8}$	$\mathbf{-3}$	$\mathbf{Log_{1/2}\sqrt{2}}$	$\mathbf{-\frac{1}{2}}$	$\mathbf{Log_{1/4}2}$	$\mathbf{-\frac{1}{2}}$
$\mathbf{Log_{1/2}\frac{1}{8}}$	$\mathbf{3}$	$\mathbf{Log_{1/3}\frac{1}{\sqrt{27}}}$	$\mathbf{\frac{3}{2}}$	$\mathbf{Log_{1/4}\sqrt{8}}$	$\mathbf{-\frac{3}{4}}$

Concentric Circles Game

Rules of the game: Each student is dealt one card. Students are divided into two groups of the same size. One group forms the inner circle, the other forms the outer circle, and pairs of players from inner and outer circle face each other. Each student in a pair takes turn to hold up their card for the other player to see. The other player must figure out the answer which is written on the back of the card. If a player gives the correct answer, they earn a point. If not, the two students in the pair figure out the solution together, and nobody earns a point. Once each pair has solved their problems, the inner circle moves two outer players to the right. On the next turn, the outer circle moves two inner players to the right, etc.

Set 1: Common and Natural logs; Change of Base.
Questions and answers for cards:

$\mathbf{Log\ 100}$	$\mathbf{2}$	$\mathbf{Log\ 10}$	$\mathbf{1}$
$\mathbf{Log\ \frac{1}{10}}$	$\mathbf{-1}$	$\mathbf{Log\ \sqrt{10}}$	$\mathbf{\frac{1}{2}}$
$\mathbf{Ln\ e}$	$\mathbf{1}$	$\mathbf{Ln\ \frac{1}{e}}$	$\mathbf{-1}$
$\mathbf{Ln\ \sqrt{e}}$	$\mathbf{\frac{1}{2}}$	$\mathbf{Ln\ 1}$	$\mathbf{0}$
$\mathbf{Log\ \frac{1}{\sqrt{10}}}$	$\mathbf{-\frac{1}{2}}$	$\mathbf{Ln\ \frac{1}{\sqrt[3]{e}}}$	$\mathbf{-\frac{1}{3}}$
Use Base 10: $\mathbf{Log_2 3 = ?}$	$\mathbf{\frac{Log\ 3}{Log\ 2} = 1.565}$	***Use Base 10:*** $\mathbf{Log_5 29 = ?}$	$\mathbf{\frac{Log\ 29}{Log\ 5} = 2.092}$
Use Base e: $\mathbf{Log_2 3}$	$\mathbf{\frac{Ln\ 3}{Ln\ 2} = 1.565}$	***Use Base e:*** $\mathbf{Log_5 29}$	$\mathbf{\frac{Ln\ 29}{Ln\ 5} = 2.092}$

Set 2: Laws of Logarithms.
Questions and answers for cards:

Expand: $Log\ mn$	**Answer:** $Log\ m + Log\ n$	**Expand:** $Log\ \frac{mn}{p}$	**Answer:** $Log\ m + Log\ n - Log\ p$
Expand: $Log\ m^3n$	**Answer:** $3Log\ m + Log\ n$	**Expand:** $Log\ \frac{\sqrt[3]{m}}{n}$	**Answer:** $\frac{1}{3}Log\ m - Log\ n$
Write a Single log $Log\ m + Log\ n - Log\ p$	**Answer:** $Log\ \frac{mn}{p}$	**Write a Single log** $Log\ m + 2Log\ n$	**Answer:** $Log\ \frac{m}{n^2}$
Write a Single log $\frac{1}{2}Log\ m + Log\ n$	**Answer:** $Log\ \sqrt{mn}$	**Write a Single log** $3Log\ m - 2Log\ n$	**Answer:** $Log\ \frac{m^3}{n^2}$
Expand: $Log_2 8mn$	**Answer:** $3 + Log_2 m + Log_2 n$	**Expand:** $Log_3 \frac{9}{mn}$	**Answer:** $2 - Log_3 m - Log_3 n$
Write a Single log $2Log\ m + 3Log\ n$	**Answer:** $Log\ \frac{m^2}{n^3}$	**Write a Single log** $1 - Log\ n$	**Answer:** $Log\ \frac{10}{n}$
Expand: $Log\ \frac{m}{10n}$	**Answer:** $Log\ m - Log\ n - 1$	**Write as a Single log** $\frac{1}{2}Log_5 p - 2Log_5 q$	**Answer:** $Log_5 \frac{\sqrt{p}}{q^2}$

Reteaching Properties of Logarithms Name: ___________________

Rewrite each problem using the change–of–base formula and then evaluate using a calculator.

1) Change to log. $\log_4 30$

2) Change to ln. $\log_5 125$

3) Change to log. $\log_{16} 2$

Rewrite each problem using the product property.

4) Expand. $\log 5x$

5) Expand. $\ln xy$

6) Condense. $\log 4 + \log p + \log t$

Rewrite each problem using the quotient property.

7) Expand. $\log \frac{7}{2}$

8) Expand. $\ln \frac{2}{m}$

9) Condense. $\log r - \log t - \log u$

Rewrite each problem using the power property.

10) Expand. $\log x^3$

11) Expand. $\ln y^{-5}$

12) Condense. $10 \log p$

Reteaching Properties of Logarithms Name: _ANSWER KEY____

Rewrite each problem using the change–of–base formula and then evaluate using a calculator.

1) Change to log. $\log_4 30$ **Solution:** $\dfrac{\log 30}{\log 4} = 2.453$

2) Change to ln. $\log_3 81$ **Solution:** $\dfrac{\ln 81}{\ln 3} = 4$

3) Change to log. $\log_{16} 2$ **Solution:** $\dfrac{\log 2}{\log 16} = .25 = \dfrac{1}{4}$

Rewrite each problem using the product property.

4) Expand. $\log 5x$ **Solution:** $\log 5 + \log x$

5) Expand. $\ln xy$ **Solution:** $\ln x + \ln y$

6) Condense. $\log 4 + \log p + \log t$ **Solution:** $\log 4pt$

Rewrite each problem using the quotient property.

7) Expand. $\log \dfrac{7}{2}$ **Solution:** $\log 7 - \log 2$

8) Expand. $\ln \dfrac{2}{m}$ **Solution:** $\ln 2 - \ln m$

9) Condense. $\log r - (\log t + \log u)$ **Solution:** $\log \dfrac{r}{tu}$

Rewrite each problem using the power property.

10) Expand. $\log x^3$ **Solution:** $3 \log x$

11) Expand. $\ln y^{-5}$ **Solution:** $-5 \ln y$

12) Condense. $10 \log p$ **Solution:** $\log p^{10}$

Laws of Logarithms Worksheet Name: ____________________

1) **Expand:** $Log_2 8mn$		2) **Expand:** $Log_3 \frac{\sqrt{3}}{mn}$	
3) **Expand:** $Log\ 10m^3\sqrt{n}$		4) **Expand:** $Log\ \frac{\sqrt[3]{m}}{n^3}$	
5) **Write a Single log** $Ln\ m + Ln\ n$ $-Ln\ p - Ln\ q$		6) **Write a Single log** $3Log\ m - 2Log\ n - \frac{1}{2}Log\ p$	
7) **Write a Single log** $\frac{1}{2}Log\ m + \frac{1}{3}Log\ n$		8) **Write a Single log** $1 + Log_2 m$	
9) **Write a Single log** $3 - Log_3 n$		10) **Write a Single log** $\frac{1}{2} - 2Log_5 k$	
11) **Expand:** $Log_4 \frac{8m}{\sqrt{2n}}$		12) **Write as a Single log** $\frac{5}{2} + Log_4 m - Log_4 n$	

Laws of Logarithms Worksheet Name: ___ANSWER KEY________

1) $3+Log_2 m+Log_2 n$	**2)** $\frac{1}{2}-Log_3 m-Log_3 n$
3) $1+3Log\ m+\frac{1}{2}Log\ n$	**4)** $\frac{1}{3}Log\ m-3Log\ n$
5) $Log\ \frac{mn}{pq}$	**6)** $Log\ \frac{m^3}{n^2\sqrt{p}}$
7) $Log\ \sqrt{m}\sqrt[3]{n}$	**8)** $Log_2 2m$
9) $Log_3 \frac{27}{n}$	**10)** $Log_5 \frac{\sqrt{5}}{k^2}$
11) $\frac{5}{4}+Log\ m-Log\ n$	**12)** $Log_4 \frac{32m}{n}$

Basic Exponential and Logarithmic Equations Name: ____________________
Round Table Game.

<u>Rules of the game</u>: Each student in a group (3–5 people) receives the worksheet. They all start by working through the first problem and then pass their worksheet to the next person clockwise. The next person checks the previous problem and does the next one, then passes the worksheet on. The process continues until all the problems on the worksheets are completed.

1) $3^x = 9$ Check: ☐	2) $\log_2 x = 3$ Check: ☐
3) $2^{3x} = 8$ Check: ☐	4) $\log_3 (x-1) = 2$ Check: ☐
5) $4^x = 8$ Check: ☐	6) $\log_2 x = -4$ Check: ☐
7) $\sqrt{5}^{4x} = 25$ Check: ☐	8) $\log_5 (3x) = \frac{1}{2}$ Check: ☐
9) $3 \cdot 9^x = \frac{1}{27}$ Check: ☐	10) $\log_9 (3x+4) = \frac{1}{2}$ Check: ☐
11) $\left(\sqrt{8}\right)^{x+1} = 16$ Check: ☐	12) $\log_{1/4} (x-1) = 0$ Check: ☐

Basic Exponential and Logarithmic Equations ANSWER KEY

1) $x = 2$	2) $x = 8$
3) $x = 1$	4) $x = 10$
5) $x = \frac{3}{2}$	6) $x = \frac{1}{16}$
7) $x = 1$	8) $\frac{\sqrt{5}}{3}$
9) $x = -2$	10) $x = -\frac{1}{3}$
11) $x = 3$	12) $x = 2$

Laws of Logarithms in Logarithmic Equations Name: ____________________

Coach and Player game.

<u>*Rules of the game*</u>: Students are paired up, and each pair receives a worksheet. Students in each pair take turns assuming Coach and Player roles. The game can take two paths: the Coach can observe and guide as the Player is solving the problem; or the Coach can solve a problem verbally while the Player writes down exactly what the Coach tells them to write (excellent way to develop students' math vocabulary and articulation of ideas). Encourage students to use positive reinforcement and constructive but positive corrections.

1) $\log x + \log 2 = \log 10$	2) $\log_8 2 + \log_8 x = \frac{1}{3}$
3) $\ln x - 2\ln 3 = 4$	4) $\ln(x-1) + 2\ln 5 = 0$
5) $\frac{1}{2}\log x - 3\log 2 + \log 20 = 1$	6) $2\log_3 x - \frac{1}{2}\log_3 16 = 2$

Laws of Logarithms in Logarithmic Equations ANSWER KEY

1) $x = 5$	**2)** $x = 1$
3) $x = 9e^4$	**4)** $x = \frac{26}{25}$
5) $x = 16$	**6)** $x = 6$

Solving Exponential and Logarithmic Equations Name: ________________

Solve each equation.

1) $7^{x} = 56$ **Think: How do you undo the 7?**

Think: What happens to the x?

Check!

2) $3^{x-5} = 10$ **Think: How do you undo the 3?**

Think: What happens to the *x*–5?

Check!

3) $\log_2 y = 4$ **Think: How do you undo the log?**

Think: What happens to the *y*?

Check!

4) $\log_4(3x+5) = 5$ **Think: How do you undo the log?**

Think: What happens to the 3*x* + 5?

Check!

Solving Exponential and Logarithmic Equations Name: _ANSWER KEY____

Solve each equation.

1)

$$7^x = 56$$

$$\cancel{\log_7 7}^x = \log_7 56$$

$$x = \frac{\log 56}{\log 7} = 2.069$$

2)

$$3^{x-5} = 10$$

$$\cancel{\log_3 3}^{x-5} = \log_3 10$$

$$x - 5 = \frac{\log 10}{\log 3}$$

$$x = 7.096$$

3)

$$\log_2 y = 4$$

$$\cancel{2}^{\cancel{\log_2} y} = 2^4$$

$$y = 16$$

4)

$$\log_4 (3x+5) = 5$$

$$\cancel{4}^{\cancel{\log_4} (3x+5)} = 4^5$$

$$3x + 5 = 1024$$

$$3x = 1019$$

$$x = 339\frac{2}{3}$$

Applications of Logarithms Name: ________________

1) The magnitude of an earthquake was defined in 1935 by Charles Richter by the expression $\boldsymbol{M} = \log \frac{\boldsymbol{I}}{\boldsymbol{S}}$ where $\boldsymbol{I}$ is the intensity of the earthquake and $\boldsymbol{S}$ is the intensity of a "regular earthquake". The magnitude of a regular earthquake is $\boldsymbol{M} = \log \frac{\boldsymbol{S}}{\boldsymbol{S}} = \log \mathbf{1} = \mathbf{0}$.

   a. If the intensity of an earthquake in Logarithville is 3 times the intensity of a regular earthquake, what is its magnitude?

   b. Early in the 20^{th} century, the earthquake in San Francisco registered 8.3 on the Richter scale. By what factor *x* of $\boldsymbol{S}$ was the intensity of this earthquake higher than that of a regular earthquake?

   c. In the same year, another earthquake in South America was recorded to be 4 times more intense than that of San Francisco. What was its magnitude?

2) The population growth problems are often modeled by the equation $\boldsymbol{N}(\boldsymbol{t}) = \boldsymbol{N}_{\mathbf{0}}\left(\mathbf{1}+\boldsymbol{r}\right)^{\boldsymbol{t}}$ where $\boldsymbol{N_0}$ = original population size, $\boldsymbol{N}(\boldsymbol{t})$ = final population size, $\boldsymbol{r}$ = rate of increase/decrease of the population, and $\boldsymbol{t}$ = time that has passed. Suppose a group of 34 rabbits increases their numbers by 40% every year.

   a. How many rabbits will there be after 3 years?

   b. How many years will it take for the population to double?

3) Radioactive decay is often modeled by the equation $\boldsymbol{A}(\boldsymbol{t}) = \boldsymbol{A}_{\mathbf{0}}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{t}/\boldsymbol{h}}$ where $\boldsymbol{A}_{\mathbf{0}}$ = the original amount of element present, $\boldsymbol{A}(\boldsymbol{t})$ = the final amount, $\boldsymbol{h}$ = half–life of the element (how long it takes for the element to decay to half of its original amount), and $\boldsymbol{t}$ = time that has passed. The radioactive substance Carbon–14, present in all living things, has a half–life of 5730 years. By measuring the amount of C–14 present in a fossil, scientists can estimate how old the fossil is. Suppose a group of archaeologists discovered what they think may be Barny's great ancestor, Barnyssaorus Rex. There seems to be 65% of the radioactive element remaining in the bones. How old is the fossil?

Applications of Logarithms Name: _ANSWER KEY____

1) a. Let the intensity of the earthquake in Logarithville be $\boldsymbol{I} = \mathbf{3}\boldsymbol{S}$. Then its magnitude, $\boldsymbol{M} = \log\frac{\boldsymbol{I}}{\boldsymbol{S}} = \log\frac{\mathbf{3}\boldsymbol{S}}{\boldsymbol{S}} = \log\frac{\mathbf{3}\cancel{\boldsymbol{S}}}{\cancel{\boldsymbol{S}}} = \log\mathbf{3} = \mathbf{0.477}$.

   b. Let $\boldsymbol{x}$ = factor of $\boldsymbol{S}$. Then the intensity $\boldsymbol{I} = \boldsymbol{xS}$, and so

$$\boldsymbol{M} = \log\frac{\boldsymbol{I}}{\boldsymbol{S}} = \log\frac{\boldsymbol{xS}}{\boldsymbol{S}} = \log\frac{\boldsymbol{x}\cancel{\boldsymbol{S}}}{\cancel{\boldsymbol{S}}} = \log\boldsymbol{x} = \mathbf{8.3}$$

   So $\log\boldsymbol{x} = \mathbf{8.3}$ and therefore, $\boldsymbol{x} = \mathbf{10}^{\mathbf{8.3}}$.

   c. $\boldsymbol{M}_{\boldsymbol{SanFrancisco}} = \mathbf{8.3} = \log\frac{\boldsymbol{I}_{\boldsymbol{SanFrancisco}}}{\boldsymbol{S}}$

$$\boldsymbol{M}_{\boldsymbol{SouthAmerica}} = \log\frac{\mathbf{4}\boldsymbol{I}_{\boldsymbol{SanFrancisco}}}{\boldsymbol{S}} = \log\left(\mathbf{4}\cdot\frac{\boldsymbol{I}_{\boldsymbol{SanFrancisco}}}{\boldsymbol{S}}\right) =$$

$$\log\mathbf{4} + \log\frac{\boldsymbol{I}_{\boldsymbol{SanFrancisco}}}{\boldsymbol{S}} = \mathbf{0.602} + \mathbf{8.3} = \mathbf{8.902}$$

2) a. For this problem, $\boldsymbol{N}_{\mathbf{0}} = \mathbf{34}$, $\boldsymbol{r}$ **= 0.4**, and $\boldsymbol{t}$ **= 3**. We want $\boldsymbol{N}(\mathbf{3})$ .Then $\boldsymbol{N}(\mathbf{3}) = \mathbf{34}\cdot(\mathbf{1} + \mathbf{0.4})^{\mathbf{3}} = \mathbf{93.296}$, or about 93 rabbits.

   b. If the population doubles, then $\boldsymbol{N}(\boldsymbol{t}) = \mathbf{2}\boldsymbol{N}_{\mathbf{0}} = \mathbf{68}$. Now $\boldsymbol{t}$ is unknown. So,

   $\mathbf{68} = \mathbf{34}\cdot(\mathbf{1} + \mathbf{0.4})^{\boldsymbol{t}}$.

   Dividing both sides by 34 yields $\mathbf{2} = \mathbf{1.4}^{\boldsymbol{t}}$.

   Taking log of both sides gives us $\log\mathbf{2} = \log\mathbf{1.4}^{\boldsymbol{t}} = \boldsymbol{t}\log\mathbf{1.4}$, and so

   $\boldsymbol{t} = \dfrac{\log\mathbf{2}}{\log\mathbf{1.4}} = \mathbf{2.06}$ years.

3) In this problem, $\boldsymbol{A}_{\mathbf{0}}$ is unknown, but since we know that 65% of the element is remaining, we can use $\boldsymbol{A}(\boldsymbol{t}) = \mathbf{0.65}\cdot\boldsymbol{A}_{\mathbf{0}}$. Half–life is $\boldsymbol{h}$ = 5730, and $\boldsymbol{t}$ is unknown.

$$\mathbf{0.65}\cdot\boldsymbol{A}_{\mathbf{0}} = \boldsymbol{A}_{\mathbf{0}}\cdot\left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{t}/\mathbf{5730}}$$

   So we get $\mathbf{0.65}\cdot\cancel{\boldsymbol{A}_{\mathbf{0}}} = \cancel{\boldsymbol{A}_{\mathbf{0}}}\cdot\left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{t}/\mathbf{5730}}$

$$\mathbf{0.65} = \left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{t}/\mathbf{5730}}$$

   Taking **log** of both sides yields $\log\mathbf{0.65} = \log\left(\frac{\mathbf{1}}{\mathbf{2}}\right)^{\boldsymbol{t}/\mathbf{5730}} = \frac{\boldsymbol{t}}{\mathbf{5730}}\cdot\log\frac{\mathbf{1}}{\mathbf{2}}$

   Solving for $\boldsymbol{t}$, we get $\dfrac{\boldsymbol{t}}{\mathbf{5730}} = \dfrac{\log\mathbf{0.65}}{\log\frac{\mathbf{1}}{\mathbf{2}}}$ and so $\boldsymbol{t} = \mathbf{5730}\cdot\dfrac{\log\mathbf{0.65}}{\log\frac{\mathbf{1}}{\mathbf{2}}} = \mathbf{3561}$ years.

Summative Assessment Name: ________________

Rewrite the logarithm in exponential form.

1) $\log_{25} 5 = x$

2) $\log_{3} 81 = m$

Rewrite the exponent in logarithmic form.

3) $4^{x} = 64$

4) $\frac{1}{2}^{m} = 4$

Evaluate the logarithms using $\log 4 = 0.602$ **and** $\log 14 = 1.146$**.**

5) $\log_{4} 14$

6) $\log \frac{14}{4}$

7) $\log 56$

8) $\log 16$

Solve the equation.

9) $8^w + 7 = 27$

10) $4^{5t} = 100$

11) $\ln 6 + \ln x = 3$

12) $\log_3 x^2 - \log_3 7 = 12$

13) Maryville was founded in 1950. At that time, 500 people lived in the town. The population growth in Maryville follows the equation $P = 500 + 1.5^t$, where *t* is the number of years since 1950.

a) Determine when the population had doubled since the founding.

b) In what year was the population predicted to reach 25,000 people?

c) What social implications could the population growth in that number of years have on the town?

Summative Assessment Name: ___Answer Key____________

Rewrite the logarithm in exponential form.

1) $25^x = 5$

2) $3^m = 81$

Rewrite the exponent in logarithmic form.

3) $\log_4 \mathrm{x} = 64$

4) $\log_{\frac{1}{2}} 4 = m$

Evaluate the logarithms using $\log 4 = 0.602$ **and** $\log 14 = 1.146$**.**

5) $\dfrac{\log 14}{\log 4} = \dfrac{1.146}{.602} = 1.904$

6) $\log 14 - \log 4 = 1.146 - 0.602 = 0.544$

7) $\log 4 \cdot 14 = \log 4 + \log 14 = 1.146 + 0.602 = 1.748$

8) $\log 4^2 = 2 \log 4 = 2 \cdot 0.602 = 1.204$

Solve the equation.

9)
$$8^w + 7 = 27$$
$$8^w = 20$$
$$\log_8(8^w) = \log_8 20$$
$$w = \log_8 20 = \frac{\log 20}{\log 8} = 1.441$$

10)
$$4^{5t} = 100$$
$$\log_4(4^{5t}) = \log_4 100$$
$$5t = \log_4 100 = \frac{\log 100}{\log 4}$$
$$5t = 3.322$$
$$t = 0.664$$

11) $\ln 6 + \ln x = 3$

$\ln 4x = 3$

$e^{\ln 4x} = e^3$

$4x = 20.086$

$x = 5.021$

12) $\log_3 x^2 - \log_3 7 = 12$

$\log_3 \frac{x^2}{7} = 12$

$3^{\log_3 \frac{x^2}{7}} = 3^{12}$

$\frac{x^2}{7} = 531441$

$x^2 = 3720087$

$x = 1928.75$

13) Maryville was founded in 1950. At that time, 500 people lived in the town. The population growth in Maryville follows the equation $\mathrm{P} = 500 + 1.5^{\mathrm{t}}$, where *t* is the number of years since 1950.

a) Determine when the population had doubled since the founding.

$\mathrm{P} = 500 + 1.5^{\mathrm{t}}$

$1000 = 500 + 1.5^{\mathrm{t}}$

$500 = 1.5^{\mathrm{t}}$

$\log_{1.5} 500 = \mathrm{t}$

$\mathrm{t} = 15.3$

The population had doubled in the year 1965.

b) In what year was the population predicted to reach 25,000 people?

$25000 = 500 + 1.5^{\mathrm{t}}$

$24500 = 1.5^{\mathrm{t}}$

$\log_{1.5} 24500 = \log_{1.5} 1.5^{\mathrm{t}}$

$\mathrm{t} = 24.9$

The population had reach 25,000 just prior to the 1975.

c) Discuss whether the population growth predicted by this equation is reasonable over a entire century. Include any social implication that could occur.

Student answers should include comparing the original population and the population in 1975 or later. Discussions could include job and housing availability, traffic implications of population boom, etc.